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Activity 2

1 Synopsis.

What this activity is about. In the previous seminar you saw how to assess estimators using the Monte Carlo method. You now have to use this technique again in conjunction with the theory, to examine the properties of several estimators.

What you need. You need to be fully acquainted with the content of previous seminars and practice assignment. You will also need access to a computer equiped with R.

2 Background

We have seen that seemingly natural candidates to estimate a location parameter can be a total failure, if the distribution does not have moments (Cauchy) or fails to verify regularity conditions (case of the $U(0,\theta)$, discussed in class). In this activity, several exercises try to highlight that:

1. There is nothing sacred about unbiased estimators (which may not even exist!). Sometimes, we may prefer a biased estimator.
2. Even if $\hat{\theta}$ is the MLE of θ and achieves the Cramér-Rao lower bound, it may not be the best estimator in terms, for instance, of MSE.

3 Problems

1. Consider $X \sim N(m = 2, \sigma = 10)$.

   (a) Generate $N = 1000$ samples of size $n = 50$, computing for each sample
   - The MLE, $\hat{m} = \overline{X}$, which you know is unbiased for m and Cramér-Rao achieving, hence "best" in a certain sense.
   - The estimator $\hat{m}_b = 0.98\overline{X}$, i.e. an "shrinked" average.

   (b) Estimate the bias for both estimators. Compare with the theoretical bias.

   (c) Estimate the MSE of both estimators. Which estimator is best in terms of MSE?

   (d) What is the Cramér-Rao lower bound for unbiased estimators in this problem?

   (e) Can $\hat{m}_b$ have a MSE *below* the Cramér-Rao lower bound for unbiased estimators?

2. Consider $X \sim U(0, \theta)$. We saw this example in class, and argued that the moment estimator of θ was in many cases easy to improve, hence not optimal.

   (a) What is the moment estimator of θ? (Hint: Check your class notes.)

   (b) What is the MLE of θ (Hint: Check your class notes.)

   (c) Generate $N = 1000$ samples of size $n = 50$ from the $U(0, \theta)$ with $\theta = 2$. For each sample compute: i) The moment estimator of θ, and ii) The MLE of θ. Save your results.

   (d) Estimate the bias of both estimators. It should be close to the theoretical values that you can obtain easily by hand.

   (e) Estimate the mean square error (MSE) of both estimators. Which one is larger?

   (f) Which estimator would you prefer?

3. What can you say about the consistency of the estimators in the two previous problems?

4 Hints and comments

1. When comparing biased estimators, variance alone is of no help; MSE (mean square error, $E(\hat{\theta} - \theta)^2$) makes more sense.

2. A posible work flow for the both problems would then be:

   (a) Create a matrix with 1000 rows and 2 columns to hold the results of $\hat{m}_1$ and $\hat{m}_2$ for each of the $N = 1000$ samples. Initially, fill it with zeros or whatever.

   ```
   > results <- matrix(0, 1000, 2)
   ```

   (b) Open a loop, `for (i in 1:1000) { }` inside which you generate the sample, compute both $\hat{m}_1$ and $\hat{m}_2$, and save them in row `i` and columns 1 and 2 of matrix `results`.

   (c) To compute $\hat{m}_1$ and $\hat{m}_2$ you may use the functions `mean` and `max`.

   (d) When you exit the loop, estimate the bias and/or the mean square error for both columns. For instance, for the first estimator you could do:

   ```
   > MSE.m1 <- mean( (results[,1]-2)^2 )
   ```

   and similarly for `MSE.m2`. Then, compare both.