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Activity 2

1 Synopsis.

What this activity is about. In the previous seminar you saw how to assess estimators using the Monte Carlo method. You now have to use this technique again in conjunction with the theory, to examine the properties of several estimators.

What you need. You need to be fully acquainted with the content of previous seminars and practice assignment. You will also need access to a computer equiped with R.

2 Background

We have seen that seemingly natural candidates to estimate a location parameter can be a total failure, if the distribution does not have moments (Cauchy) or fails to verify regularity conditions (case of the $U(0, \theta)$, discussed in class). In this activity, several exercises try to highlight that:

1. There is nothing sacred about unbiased estimators (which may not even exist!). Sometimes, we may prefer a biased estimator.
2. The Cramér-Rao is not applicable in the form studied to biased estimators, which may have lower variance that the CR lower bound.

3 Problems

1. Consider $X \sim N(m = 2, \sigma = 10)$.

   (a) Generate $N = 1000$ samples of size $n = 50$, computing for each sample
   - The MLE, $\hat{m} = \overline{X}$, which you know is unbiased for m and Cramér-Rao achieving, hence "best" in a certain sense.
   - The estimator $\hat{m}_b = 0.98\overline{X}$, i.e. an "shrinked" average.

   (b) Estimate the bias for both estimators. Compare with the theoretical bias.

   (c) Estimate the MSE of both estimators. Which estimator is best in terms of MSE?

   (d) What is the Cramér-Rao lower bound for unbiased estimators in this problem?

   (e) Can $\hat{m}_b$ have a MSE *below* the Cramér-Rao lower bound for unbiased estimators?

   (f) Are some, or all, of the above estimators depoendent on sufficient statistics?

2. Consider a sample from a distribution $N(\theta, 1)$. You are required to estimate θ and it is known that $\theta \geq 2$.

   (a) What form would have the likelihood function associated to a sample of n observations?

   (b) What would be the $\hat{\theta}_{MLE}$ if $\overline{x} = 2.85$?

   (c) What would be the $\hat{\theta}_{MLE}$ if $\overline{x} = 1.93$?

   (d) Assume the true value of θ were just at the boundary, $\theta = 2$. What would be the probability of getting nonetheless a value of $\overline{X}$ smaller than 2?

   (e) In the light of the previous answer, what would be the probability of $\hat{\theta}_{MLE} = 2$ when $\theta = 2$? Is the distribution of $\hat{\theta}_{MLE}$ asymptotically normal, as is usually the case with the MLE?

   (f) Can we use the Cramér-Rao lower bound to compute the best attainable variance for an estimator of θ in this case? Why or why not?

   (g) Simulate $N = 1000$ samples of size $n = 20$ from the distribution $N(\theta = 2, 1)$, compute the MLE and compute its bias and variance across the N samples. Ordinarily, when estimating the mean of a normal, the CR lower bound is σ^2/n; what do you see in your simulation? Explain.

   (h) Is $\hat{\theta}_{MLE}$ consistent?

3. What can you say about the consistency of the estimators in the two previous problems?