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Seminar 2

1   Synopsis.

What we are set about to do.   This seminar will set up the stage so that you can easily tackle Activity 2.

What you need to know.   In order to benefit from this activity you only need to know some rudiments of R. The introduction in the computer practice session already held may well be enough. Go back to what you did in Activity 1 and remember the use of instructions such as `runif`, `if` and `for`.

2   Background.

2.1   Financial options.

A *call option* is a financial instrument which gives its owner the right (but not the obligation) to buy a certain asset (the *underlying asset*; usually shares of a publicly traded company, fixed quantities of precious metals or other commodities, etc.) at a certain price (the *strike price*) at a given moment in the future (the *expiration date*)[1].

A *put option* is described likewise, only the right of the owner is now to sell an asset at the expiration date[2].

We will consider an European option (i.e., can only be exercised ad the expiration date) under a simplified (and somewhat unrealistic) set of assumptions.

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[1]There are options which give the owner the same right at any time between the present and the expiration date. We will assume an *European option* which can only be exercised at the time it expires.

[2]This summary description is sufficient for our purposes. You will have occasion to learn more about Finance; for now, you may turn to Google, to the Wikipedia, http://en.wikipedia.org/wiki/Option_(finance) or textbooks such as [1].

1. The price of the underlying asset can only change each period by either climbing 1€, falling 1€ or staying the same, i.e. the only possible changes in the price are +1, 0 and −1.

2. There remain 48 periods until expiration.

3. At each of these periods, the probability of the price of the underlying asset climbing 1€ is 0.50, the probability of falling by 1€ is 0.45, and the probability of staying the same is 0.05.

4. The price movement at each period is independent of what happens in any other period.

5. The price of the underlying asset now is 55€. The strike price after the 48 periods until expiration is 53€.

2.2   Option valuation

Option valuation in the real world is a complex affair. We will settle for the much simpler task of finding the expected value now of the simplified option outlined above at expiration time.

Notice that this depends solely on the price of the underlying asset. If the underlying asset is worth more than the strike price, the option will be valuable, otherwise it will be worthless.

One easy way to obtain an approximation of the distribution of prices of the underlaying asset 48 periods from now is simulation. We can simulate many, say $N = 10000$ trajectories, and note how often we get a given price at the end of the 48 periods. Look at the following code:

```
> NofRuns  <- 10000
> Periods  <- 48
> PriceNow <- 55
> Strike   <- 53
> Freqs    <- rep(0,PriceNow+Periods)
> Prices   <- 1:length(Freqs)
> for (i in 1:NofRuns) {
    price <- PriceNow
    for (j in 1:Periods) {
      r <- runif(1)
      if (r > 0.50) {
        price <- price + 1
      }
      if (r < 0.45)
        price <- price - 1
    }
    Freqs[price] <- Freqs[price] + 1
  }
```

When we reach the end of the outer loop, `Freqs` is an array which contains in the i-th cell the number of times the observed price at the end of the period was i. We can plot the results:

```
> names(Freqs) <- Prices
> barplot(Freqs,xlab="Prices",ylab="Frequencies")
```
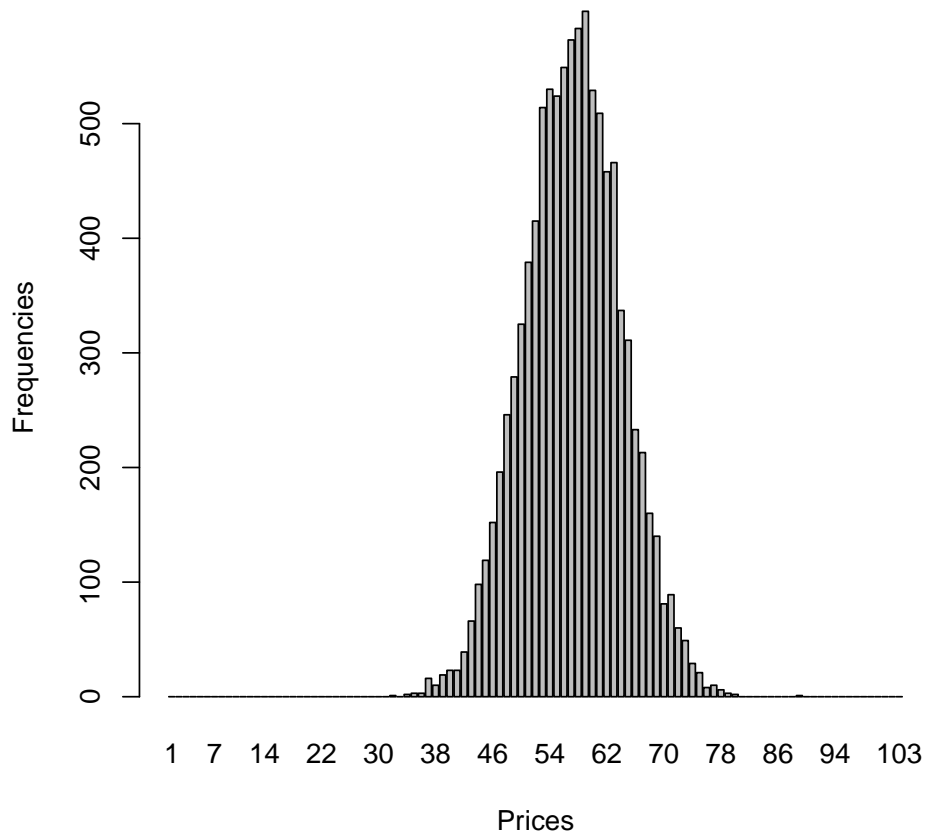

We can inspect `Freqs` (only some entries), compute relative frequencies and an approximation to the expected price of the underlying asset:

```
> Freqs[50:60]

 50   51   52   53   54   55   56   57   58   59   60
325  379  415  514  530  524  549  573  583  598  529

> RelFreqs <- Freqs / NofRuns
> ExpectedPrice <- sum( Prices * RelFreqs  )
> ExpectedPrice
```

```
[1] 57.2511
```

(Compare with the theoretical result, that you can easily calculate!) The expected price of the option at the end of the 48 periods is approximated by:

```
> Profit         <- Prices - Strike
> Profit         <- ifelse(Profit < 0, 0, Profit)
> ExpPriceOpt    <- sum( Profit * RelFreqs  )
> ExpPriceOpt
```

```
[1] 5.3579
```

3   Some remarks

A few comments on the code presented are:

1. We have used the fact that a uniform random variable between 0 and 1 takes value greater than 0.5 with probability 0.5; a simple way to simulate an event with probability 0.5 is then to generate a uniform r and then check if it is greater than 0.5.

   A simpler way of generating the changes in price with the probabilities given, would be in our particular case:

   ```
   > NofRuns  <- 10000
   > Periods  <- 48
   > PriceNow <- 55
   > Strike   <- 53
   > Freqs    <- rep(0,PriceNow+Periods)
   > Prices   <- 1:length(Freqs)
   > for (i in 1:NofRuns) {
       price <- PriceNow
       for (j in 1:Periods) {
         price <- price + sample(x=c(-1,0,1), size=1, prob=c(0.45,0.05,0.50))
       }
       Freqs[price] <- Freqs[price] + 1
     }
   ```

   Check the instruction `sample` to understand what it does.

2. The original code presented is also inefficient in that it checks whether $r > 0.50$ and *then* checks again whether $r < 0.45$; if the first is true, we might avoid the second test, which is certain to fail. Check the instruction `if ...  else ...` to see how you might go about doing this.

References

[1] J.M. Marín and G. Rubio. *Economía Financiera*. Bosch, Barcelona, 2001.