Handout 6

1. The waiting time for a patient in need of surgery at a given hospital is exponentially distributed, with mean $m = 20$ days.

   (a) What is the probability that he will have to wait more than 15 days?

   (b) What is the probability that he will have to wait between 15 an 30 days?

2. Let X_1 and X_2 be independent, with respective distributions $\gamma(4, 2)$ and $\gamma(3, 1)$.

   (a) What is the distribution of $Y = 8X_1 + 6X_2$?

   (b) Compute $P(Y \leq 14.4)$.

3. Let X be distributed as $\gamma(5, 1)$. What is the distribution of $Y = 10X$? What is the probability $P(Y \leq 4.61)$?

4. Let $X \sim t_5$. What is the probability $P(X^2 \geq 6.61)$?

5. Atoms of radioactive elements disintegrate randomly. The time until disintegration of an atom counting from now has density $f(t) = \lambda e^{-\lambda t}$. The so-called *half-life* is the time elapsed until 50% of the atoms now present have disintegrated; some elements decay fairly quickly, others have a half-life of hundreds or thousands of years.

   (a) If time is measured in years and for a given radioactive material $\lambda = 0.2$, how long is its half-life?

   (b) Is the half-life equal to the mean life of an atom? To some other location statistic?

6. Let $X_1 \sim N(5, \sigma^2 = 9)$, $X_2 \sim N(4, \sigma^2 = 4)$ and $X_3 \sim N(3, \sigma^2 = 1)$ be independent of each other.

   (a) If we define the random variable,

   $$Y = \frac{\sqrt{2}(X_3 - 3)}{\sqrt{\left(\frac{X_1 - 5}{3}\right)^2 + \left(\frac{X_2 - 4}{2}\right)^2}},$$

   what is its distribution?

   (b) If

   $$Z = \left(\frac{X_1 - 5}{3}\right)^2 + \left(\frac{X_2 - 4}{2}\right)^2 + (X_3 - 3)^2$$

   compute $P(2.37 < Z < 6.25)$.

7. There are 10 children in a given family. Assuming that boys are just as likely as girls, ($p = 0.50$), compute:

   (a) The probability that five children are boys and five are girls.

   (b) The probability that there are at least three and no more than seven boys among the ten children.

   (Rozanov (1977), p. 65, prob. 2)

8. Let $X_1 \sim \gamma(a_1, r_1)$ an $X_2 \sim \gamma(a_2, r_2)$ independently of each other. Give a sufficient condition on c_1 and c_2 such that:

$$c_1 X_1 + c_2 X_2 \sim \gamma(a, r).$$

   (Ash (2011), p. 14, prob. 2)

9. Suppose the probability of hitting a target with a single shot $p = 0.001$. What is the probability of hitting the target 2 or more times in 5000 shots?

   (Rozanov (1977), p. 65, prob. 3)

10. The page proof of a 500-page book contains 500 misprints randomly distributed over the whole book. What is the probability, if we select a page at random, that it contains 2 or more misprints?

    (adapted from Rozanov (1977), p. 65, prob. 4)

11. Obtain the quantile leaving to its *left* a probability of 0.10 in the $\mathcal{F}_{4,6}$ distribution.

References

R. Ash. *Statistical Inference a Concise Course*. Dover Publications Inc., United States, 2011. ISBN 9780486481586.

Y. Rozanov. *Probability Theory: A Concise Course*. Dover Publications, 1977.

Pautas docentes

1. Ejercicios de rutina, no de realización común y sustituibles por cualesquiera otros similares.

2. En el ejercicio 1 se propone la distribución exponencial como modelo de un tiempo de espera. Ya vimos que la distribución exponencial carece de memoria y, estrictamente, es difícil que un tiempo de espera sea exponencial. Ello no quita para que pueda ser un modelo adecuado en muchos casos: es un caso análogo al de la aproximación de una binomial por una normal, que puede tomar valores entre $-\infty$ y $+\infty$, inalcanzables por una binomial.

3. Los problemas 7 y 9 pueden dar lugar a mencionar la distribución binomial negativa (la distribución del número de intentos para obtener un resultado prefijado), aunque no está incluida en el programa y no es materia exigible (pero proporciona ejemplos útiles más adelante).

   La v.a. "número de intentos para obtener k resultados cuando la probabilidad de un resultado es p" tiene función de probabilidad,

   $$P(Z = n) = p\binom{n-1}{k-1}p^{k-1}(1-p)^{n-k}$$

   (="probabilidad de obtener precisamente $k-1$ resultados en los primeros $n-1$ intentos y uno adicional en el intento n-ésimo.")